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Design and Optimization of an Erbium-Doped Fiber Amplifier in Amplified Lightwave System

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Abstract

Analytical solutions of homogeneously broadened two-level systems for erbium-doped fiber amplifiers pumped in 980 and 1480nm absorption band have been derived, for co-propagate pump signal only, from EDFA rate equations and light propagation equations in steady-state case. By using these deduced expressions and numerical simulated methods, important features like gain and pump threshold power have been analyzed. It has been found that for a particular pump power or carrier concentration there is an optimum amplifier length. For 980 and 1480nm pump signal maximum gain of 20 and 22dB is obtained at 100 and 120mW pump power for 30 and 50m length respectively. The maximum gain (12 and 15dB) obtained for carrier concentration ($6x10^{24}/m^3$ and $4x10^{24}/m^3$) is nearly at 20 and 40m for 980 and 1480nm pump signal respectively at optimizations. Also, the pump threshold, for significant pump-to-signal conversion, increases with fiber length and dopant concentration. The simulation has been done by using Matlab.

Keywords: Erbium-doped fiber amplifier, rate equation, gain, pump threshold, doping concentration, light propagation equation.

1. Introduction

The most popular doped fiber amplifier is an erbium doped fiber amplifier (EDFA) which is currently utilized in many long-haul lightwave systems. This amplifier is doped with rare earth ions such as erbium (Er^{3+}) which are excited to a higher level by laser pumping, resulting in a signal gain in the 1500nm wavelength window [1]. Similarly, ions such as neodymium (Nd^{3+}) or praseodymium (Pr^{3+}) are pumped with laser sources to achieve amplification in the 1300nm wavelength region [2]. Since long distance communication links typically use wavelength near 1550nm as this is the attenuation minimum of glass fiber, erbium doped fiber amplifiers (EDFAs) are most useful for long distance communications applications [3].

Currently, most of the commercial EDFAs operate in the C band, but the amplification bandwidth has been extended to include the L and S bands in recent years thus enhancing the transmission bandwidth [4]. An EDFA can be built using a single or dual pump lasers. A pumping signal can co-propagate with an input signal or it can counter-propagate. The most efficient pump band is 980nm for pre-amplifier applications since it requires much less pump power to obtain the same noise performance compared to 800nm and 1480nm pump bands [5]. On the other hand, for booster applications, a 1480nm pump is more efficient than 800 and 900nm pump bands [5].

The use of insufficient pump power or too long fiber can lead to part of amplifier providing loss rather than gain. Alternatively, increasing pump power arbitrarily or using too short a fiber leads to insufficient use of pump power. There is, therefore, an optimum fiber length that has a strong dependence on pump power, input signal power, amount of rare-earth doping, pumping wavelength [6].

The EDFA characteristics such as gain, saturation power and spontaneous amplifier noise play a key role in determining the performance of lightwave link. In theory, the gain and saturation characteristics of an EDFA can be determined from the rate equations, which in general involve solving the coupled differential equations using numerical methods [7].

2. Physical Modeling of EDFA

The energy level diagram of Er-ion is shown in Fig. 1 [8]. When pumped at 980- or 1480-nm, the EDFA acts as a three level system. Assuming that both the signal and the pump intensity distributions are separable in the active fiber, I_S and I_P can be written as –

$$I_P(r,\Phi,z) = f_P(r,\Phi)I_P(z) \quad (1)$$

$$I_S(r,\Phi,z) = f_S(r,\Phi)I_S(z) \quad (2)$$

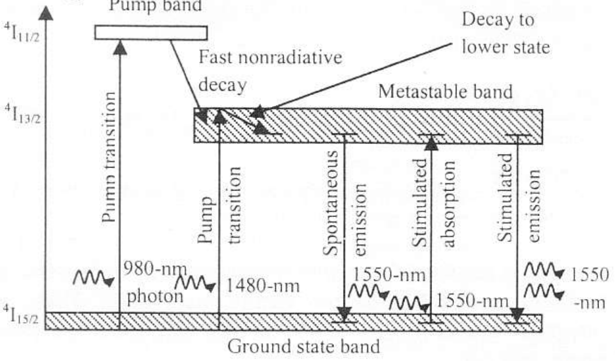

Fig. 1: Simplified energy-level diagrams and various transition process of Er3+ ions in silica.

where f_P and f_S represent the pump and signal transverse intensity profiles, respectively, normalized in such a way that –

$$\int_0^{2\pi} d\phi \int_0^{\infty} f_{P,S}(r,\phi)\, r\, dr = 1 \qquad (3)$$

In a similar way, the population density can be written as –

$$N_i(r,\phi,z) = g(r,\phi)\, N_i(z) \qquad (4)$$

where $g(r,\Phi)$ is the normalized doping intensity profile.

Introducing the overlapping integral factors Γ_i between the fields and the dopant of the semiconductor laser –

$$\Gamma_{P,S} = \int_0^{2\pi} d\phi \int_0^{a} f_{P,S}(r,\phi)\, g(r,\phi)\, r dr \quad (5)$$

where a is Er^{3+} -ion doping radius.

The rate equation describes the effects of absorption, stimulated emission and spontaneous emission on the populations of the ground and meta-stable levels. The rate equations for the EDFA which is described in an (r,Φ,z) – cylindrical coordinate system with z axis as the fiber axis can be written as [9] –

$$\begin{aligned}\frac{dN_2(r,\phi,z,t)}{dt} = {} & \frac{\sigma_{ap} I_p(r,\phi,z)}{h\nu_p} N_1(r,\phi,z,t) + \frac{\sigma_{as} I_s(r,\phi,z)}{h\nu_s} N_1(r,\phi,z,t) \\ & - \frac{\sigma_{es} I_s(r,\phi,z)}{h\nu_s} N_2(r,\phi,z,t) - A_{21} N_2(r,\phi,z,t)\end{aligned} \qquad (6)$$

$$N_1(r,\phi,z,t) + N_2(r,\phi,z,t) = N_0(r,\phi,z,t) \qquad (7)$$

where, N_1 and N_2 represent the population density of the ground level and the meta stable level, respectively; ν_p and ν_s are the pump and the signal frequencies, respectively; h is Plank's constant; σ_{as} and σ_{es} denote the stimulated absorption and emission cross sections between the ground and the meta stable state, respectively; σ_{ap} is the absorption cross section from the ground state to the pump level; $A_{21}=1/\tau_{21}$ is the spontaneous emission rate, and N_0 is the total Er^{3+}-dopant ion concentration.

Assuming that the pump light and signal light travel in the same direction along with z-coordinate, the propagation equations of the beams through the fiber with loss neglected are as follows [9] –

$$\frac{dI_p(r,\phi,z)}{dz} = -\,\sigma_{ap} N_1(r,\phi,z,t)\, I_p(r,\phi,z) \qquad (8)$$

$$\frac{dI_s(r,\phi,z)}{dz} = [\sigma_{es} N_2(r,\phi,z,t) - \sigma_{as} N_1(r,\phi,z,t)]\, I_s(r,\phi,z) \qquad (9)$$

By setting the time derivative in equation (6) to be zero, the problem is reduced to the steady state case. Then, we integrate equations (6) to (9) across the fiber transverse plane, and get –

$$\begin{aligned}0 = \frac{dN_2(z,t)}{dt} = {} & \frac{\sigma_{ap}\Gamma_p P_p(z)}{A_c h\nu_p} N_1(z,t) + \frac{\sigma_{as}\Gamma_s P_s(z)}{A_c h\nu_s} N_1(z,t) \\ & - \frac{\sigma_{es}\Gamma_s P_s(z)}{A_c h\nu_s} N_2(z,t) - A_{21} N_2(z,t)\end{aligned} \qquad (10)$$

$$N_1(z) + N_2(z) = N_0 \qquad (11)$$

$$\frac{dP_p(z)}{dz} = -\Gamma_p \sigma_{ap} N_1(z) P_p(z) \qquad (12)$$

$$\frac{dP_s(z)}{dz} = \Gamma_p[\sigma_{es} N_2(z) - \sigma_{as} N_1(z)] P_s(z) \qquad (13)$$

where, Pp(z) and Ps(z) are the pump power and the signal power at position z in the fiber; A_c represents the efficient core area for Er^{3+}-doped fiber.

Equation (10) can be rewritten as,

$$N_2(z) = \frac{\tau\Gamma_p \sigma_{ap} P_p(z)}{A_c h\nu_p} N_1(z) + \frac{\tau\Gamma_s[\sigma_{as} N_1(z) - \sigma_{es} N_2(z)] P_s(z)}{A_c h\nu_s} \qquad (14)$$

where τ is equal to τ_{21}.

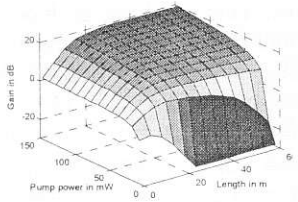

Fig. 2: Gain of EDFA dependence on length and pump power with constant doping concentration of $2.0X10^{24}m^{-3}$ (pump signal of 980nm and source signal power of 1.0mW)

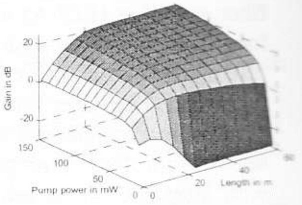

Fig. 3: Gain of EDFA dependence on length and pump power with constant doping concentration $2.0X10^{24}m^{-3}$ (pump signal power of 1480nm and source signal power of 1.0mW)

Then, eqn. (12) $\times \dfrac{\tau}{A_c\, h\nu_p}$ + eqn. (13) $\times \dfrac{\tau}{A_c\, h\nu_s}$ + eqn. (14) has been made and the obtained result is

$$N_2 + \frac{\tau}{h\nu_p}\frac{dP_p(z)}{dz} + \frac{\tau}{h\nu_s}\frac{dP_s(z)}{dz} = 0 \quad (15)$$

Assuming – $S = \int_0^z N_2(z)\, dz$,

we have –

$$S=\frac{\ln\frac{P_p(z)}{P_p(0)}+\Gamma_p\sigma_{ap}N_0z}{\Gamma_p\sigma_{ap}}=\frac{\ln\frac{P_S(z)}{P_S(0)}+\Gamma_S\sigma_{as}N_0z}{\Gamma_S(\sigma_{as}+\sigma_{es})}=-\frac{\tau}{A_c\,h\nu_P}[P_P(z)-P_P(0)]-\frac{\tau}{A_c\,h\nu_S}[P_S(z)-P_S(0)]\qquad(16)$$

The amplification is defined as $G(z) = P_s(z)/P_s(0)$.

Now, from eqn. (16) the indirect analytical solution for the amplifier gain is –

$$[G(z)]^{\beta}\exp(-\beta\Gamma_S\sigma_{es}N_0z)=1-\frac{\nu_PP_S(0)}{\nu_SP_P(0)}[G(z)-1]-\frac{[\ln G(z)+\Gamma_S\sigma_{as}N_0z]A_c h\nu_P}{\tau P_P(0)\Gamma_S(\sigma_{as}+\sigma_{es})}\qquad(17)$$

where $\beta=\dfrac{\Gamma_p\,\sigma_{ap}}{\Gamma_s\,(\sigma_{as}+\sigma_{es})}$

The pump threshold is the power required to bleach a given fiber length, which means that when the EDFA is pumped at pump threshold, the signal remains unchanged after traveling the fiber, that is, G(L)=1. Substituting it in eqn. (17) we get –

$$P_{pth}=\frac{A_c\,h\nu_P\,\sigma_{as}\,N_0\,L}{\tau\,(\sigma_{as}+\sigma_{es})\,[1-\exp(-\beta\Gamma_S\,\sigma_{es}\,N_0\,L]}\qquad(18)$$

where L is the fiber length.

3. Simulation and Result

For simulation the parameters used are – $h=6.626X10^{-34}$J.s, $\gamma_p=3.061X10^{14}$Hz (λ_p=980nm) and $\gamma_p=2.207X10^{14}$Hz (λ_p=1480nm), $\gamma_s=1.935X10^{14}$Hz (λ_s=1550nm), $\Gamma_p=\Gamma_s=0.6$, $N_0=2.0X10^{24}m^{-3}$, $A_c=1.26X10^{-11}m^2$ (a=2.0μm), 1.0μW for small launching power and 1.0mW for large launching power.

The following parameters are taken from ref. [10] for Al/Ge/SiO$_2$ fiber – τ=11.4ms, $\sigma_{ap}=3.8X10^{-25}m^2$, $\sigma_{as}=3.1X10^{-25}m^2$ and $\sigma_{es}=2.7X10^{-25}m^2$.

The amplifier gain increases and becomes saturated with pump power. The smaller the length the lower the pump power at which saturation occurs. This is because in a shorter or lower doping fiber, there are less Er^{3+} ions, thus total population inversion is obtained at lower pump power and the gain saturation occurs more easily. Here the gain saturation is caused by the pump power rather than signal power. For pump power 50, 100 and 150mW maximum gain (12, 19 and 20dB) is obtained at 20, 30 and 60m length for 980nm pump signal. For 1480nm pump signal, maximum gain (18, 20 and 22dB) is obtained at 20, 30 and 60m length for pump power 50, 100 and 150mW. Also the Fig. 2 and Fig. 3 reveal that a 1480nm pump yields a higher gain than the 980nm pump for the 1550nm signal, because the pump can maintain the necessary inversion level though the 980nm pump has the higher inversion capability. Almost similar results were found by Ali S. et al. [12] and Mahad F.D.B. et al. [13] in their works with 1550nm pump signal.

The gain increases to the maximum value, and then decreases along with the fiber, which means that the gain has a nonlinear relation to fiber length. The fiber length for which the gain is maximized is the optimum amplifier length. For 20, 40 and 60m length maximum gain 15dB is obtained at carrier concentration $6x10^{24}$, $4x10^{24}$ and $2x10^{24}m^{-3}$ respectively for both 980 and 1480nm pump signal. Also the higher the Er^{3+}-ion dopant concentration, the shorter the optimum amplifier length will be. The optimum amplifier length is shorter and the gain with length optimized under short wavelength pump power is lower than that under long wavelength pump power for the same other parameters. This result is supported by Aljaff M.P. et al. [14].

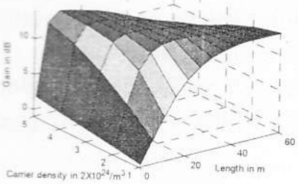

Fig. 4: Gain of EDFA dependence on length and carrier density with constant pump power of 50mW (pump signal power of 980nm and source signal power of 1.0mW)

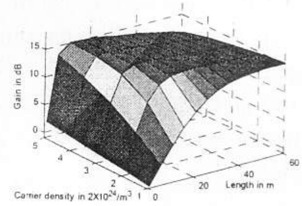

Fig. 5: Gain of EDFA dependence on length and carrier density with constant pump power of 50mW (pump signal power of 1480nm and source signal power of 1.0mW)

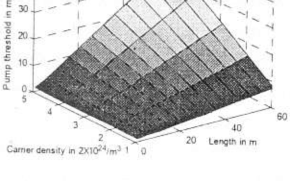

Fig. 6: Pump threshold as a function of length and carrier concentration with constant pump power of 50mW (pump signal power of 980nm and source signal power of 1.0mW)

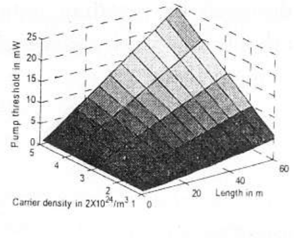

Fig. 7: Pump threshold as a function of length and carrier concentration with constant pump power of 50mW (pump signal power of 1480nm and source signal power of 1.0mW)

The pump threshold increases with fiber length and Er^{3+}-ion dopant concentration. This is because when fiber becomes longer or Er^{3+}-doping concentration becomes higher, the signal absorption becomes stronger. For 980nm pump signal maximum pump threshold of 38mW is obtained at carrier density $10\text{x}10^{24}\text{m}^{-3}$ for 60m length fiber. But for 1480nm pump signal maximum pump threshold of 24mW is obtained for the same parameters. That is, the pump threshold is much higher under pump power with short wavelength than that under pump power with long wavelength for the same other parameters.

4. Conclusion

Based on rate and propagation equations without considering fiber losses, a series of equations describing homogeneously broadened two-level system for EDFA with co-propagate pump signal have been derived. In steady-state, an indirect analytical solution for amplifier gain and a direct analytical solution for pump threshold are obtained, and are calculated using numerical simulated methods.

The results show that for all fiber lengths there is a pump threshold below which there is no significant pump-to-signal conversion is obtained. For a given pump power, there exists an optimum amplifier length for the maximum gain. The maximum gain obtained for pump powers (50, 100 and 150mW) is nearly (12, 19 and 20dB) for 980nm pump signal and (18, 20 and 22dB) for 1480nm pump signal.

The optimum fiber length decreases when erbium ion concentration increases. So it is possible to design amplifiers with high gain for amplifier length as short as few meters by increasing erbium ion concentration and vice versa. Maximum gain obtained for lengths (10, 20 and 50m) is at carrier concentration ($8\text{x}10^{24}$, $6\text{x}10^{24}$ and $2\text{x}10^{24}\text{m}^{-3}$).

According to the result at optimization EDFA can be designed by inserting optimum length with the value of erbium ion density in which gain is maximum at different pump powers.

Reference

1. Desurvire, E., 1993, Erbium-doped Fiber Amplifiers, New York: John Wiley & Sons.
2. Whitley, T.J., 1995, "A review of recent system demonstrations incorporating 1.3μm praseodymium-doped fiber amplifiers", Journal of Lightwave Technology, Vol.13, No.5, pp. 744-760.
3. DeCusatis, C.M. and DeCusatis, C.J.S., 2006, Fiber Optic Essentials, Academic Press, USA.
4. Arbore, M., et al., 2003, "S-band erbium doped fiber amplifiers for WDM transmission between 1488 and 1508nm", Optical Fiber Communication Conference, Atlanta, WK2, pp. 374-376.
5. Bjarklev, A., 1993, Optical Fiber Amplifiers, Boston: Artech House Inc.
6. Franz, J.H. and Jain V.K., 2000, Optical Communications: Components and Systems, Narosa Publishing House, India.
7. Pfeiffer, T. and Bulow H., 1992, "Analytical gain equation for erbium-doped fiber amplifiers including mode field profiles and dopant distribution", Photonics Technology Letters, vol. 4, No. 5, pp. 449-451.
8. Keiser, G., 2000, Optical Fiber Communications, McGraw-Hill Inc.
9. Geguo, DU and Guofu C, 1999, "Gain performances of 980 nm-pumped erbium-doped fiber amplifiers", Science in China (Series A), vol. 42, No. 3, pp. 286-292.
10. Pederson, B., Bjarklev, A., Lumholt, O., et al., 1991, "Detailed design of erbium-doped fiber amplifiers", IEEE Photonic Technology, vol. 3, No. 6, pp. 548.
11. Part-Enander, E. and Sjoberg, A., 1996, The Matlab handbook, Addison Wesley Longman.
12. Ali S. Khalid A.S., Al-Khateeb, Bouzid B., 2009, "A new Erbium doped fiber amplidier", Journal of Applied Sciences, vol. 9, No. 13, pp. 2498-2500.
13. Mahad F.D.B., Supa'at A.S.B.M., 2009, "EDFA gain optimization for WDM system", Elektrika, vol. 11, No. 1, pp. 34-37.
14. Aljaff P.M., Rasheed B.O., 2008, "Design optimization for efficient Erbium-doped fiber amplifiers", World Academy of Science, Engineering and Technology, vol. 46, pp. 40-43.