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Simulation of the Mathematical Modeling of a Double-Layer Capacitor Using PSIM Software

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Abstract

The double-layer capacitor (DLC) is a low voltage device exhibiting an extremely high capacitance value in comparison with other capacitor technologies of similar physical size. It's also a promising device for certain power electronic application as energy storage. A three RC branch equivalent circuit was used to characterize its terminal behavior and the equivalent circuits result are simulated using PSIM to provide the power electronics engineers a model for the terminal behavior of the DLC. An elaborated method to identify the circuit parameters was also presented here. The measurements of carbon-based DLC's showed that the simulated equivalent circuit response is similar with the experimental results.

Keywords: Double-layer capacitor, Energy storage, Three RC branch equivalent circuit, Terminal behavior, carbon-based DLC.

1. Introduction

Double-layer capacitor can be used for energy storage and peak power control in order to increase the efficiency and the life cycle of a system. Potential applications are seen at the moment in short time uninterrupted power supplies (UPS) and peak load shaving in combination with batteries [1-2]. The power density of these capacitors is higher than that of batteries, and the energy density is 10 to 20 times higher than that of electrolytic capacitors for power applications [2]. Smaller DLC's have been known for several years, but double-layer capacitors for power applications are just emerging. This paper concentrates on Carbon-Based Double-Layer Capacitors (DLC's) suitable for power applications, which although expensive, are becoming now commercially available. These capacitors are low voltage devices with a rated voltage of 2.3 V and are available with capacitance values of 470F, 900F and 1500F [3]. Higher voltages can be achieved by connecting many cells in series like in batteries. To study possible applications, a terminal model describing the behavior of the DLC is required. A three RC branch equivalent [3] is simulated to describe the terminal behavior of a DLC since the simple RC circuit is insufficient to describe nonlinearity of charge-discharge profile of a DLC. The simulated model follows more precisely the measured terminal behavior of the DLC. Physical reasoning supports the structure of the equivalent model. A method to identify the parameters of the simulated model is presented. The results are presented to gain a better understanding of the terminal characteristics of the DLC.

2. DLC Equivalent Circuit

The purpose of the equivalent circuit is to provide a model of the terminal behavior of the DLC in power electronics circuits [3-5]. The DLC consists of activated carbon particles that act as polarizable electrodes. These particles strongly packed are immersed in an electrolytic solution forming a double-layer charge distribution along the contact surface between carbon and electrolyte [3]. Three major aspects of the physics of the double-layer charge distribution have an influence in the structure of the equivalent circuit model and these are as follows.

First, based on the electrochemistry of the interface between two materials in different phases, the double-layer charge distribution of differential sections of the interface is modeled as an *RC* circuit where resistance is for the resistivity of the carbon particles and the capacitor is due to the capacitance between carbon and electrolyte.

Second, based on the theory of the interfacial tension in the double-layer, the capacitance of the double-layer charge distribution depends on the potential difference across the material.

Third, the double-layer charge distribution shows certain self discharge.

The choice of three branches is the least number, if good accuracy is wanted for the specified time range of 30 min. Each of the three branches has a distinct time constant differing from the others in more than an order of magnitude which will result in an easily measurable model. The first or immediate branch, with the elements R_i, C_{io} and the voltage- dependent capacitor C_{i1} (in F/V), dominates the immediate behavior of the DLC in the time range of seconds in response to a charge action. The second or delayed branch, with parameters R_d and C_d dominates the terminal behavior in the range of minutes. Finally, the third or long-term branch, with parameters R_l and C_l determines the behavior for times longer than 10 min [3].

A leakage resistor, parallel to the terminals, is added to represent the self discharge property [3]. The proposed equivalent circuit is shown in Fig.1. (→).

3. Parameters Identification

As stated before, the physics of the DLC predicts a voltage dependent value of capacitance. To simplify the model, this property has only been assigned to the first or immediate branch in the simulated DLC model. The usual linear definition of capacitance is the following:

$$C = \frac{Q}{V} \quad \ldots\ldots\ldots\ldots(1)$$

with Q the stored charge and V the capacitor voltage. The same definition of applies if the charge Q is the total charge in the device or an incremental charge ΔQ resulting from an incremental change ΔV in voltage. This definition is not valid for voltage-dependent capacitance. A useful definition, as it describes the change in charge at a given voltage, is the differential capacitance. This capacitance is defined as

$$C_{diff}(V) = \frac{dQ}{dV}\Big|_{V'} \ldots\ldots\ldots\ldots(2)$$

with dQ an incremental change in charge at a certain capacitor voltage V' that produces an incremental change in voltage dV.

Based on the physics of the double layer and the usable range of voltage for the DLC, the differential capacitance is modeled as a constant capacitor and a capacitor which value varies linearly with the voltage

$$C_{diff}(V) = C_{i0} + C_{i1} * V \ldots\ldots\ldots\ldots(3)$$

To verify this model of the nonlinear capacitance, measurements have been carried out to determine the parameters C_{i0} and Ci1. This nonlinear behavior of the immediate capacitance of the DLC has the consequence that more energy per voltage increment is stored at higher voltage than it would be in a constant linear capacitor. Calculating the stored energy in the immediate branch of the capacitor, it is

$$E = \frac{C_{i0}}{2} \times V^2 + \frac{C_{i1}}{3} \times V^3 \ (=1/2 C_e V^2) \ldots\ldots\ldots\ldots(4)$$

This is derived by using (2) and (3) and a constant charge current I, from $E = \int V \times i dt \ldots\ldots\ldots\ldots (5)$ with $i dt = I dt = dQ = C_{diff}\, dV$ and $E = \int (C_{i0} + C_{i1} \times V) \times V \times dV$

First, the equivalent capacitance C_q defined as the capacitance value of a linear capacitor holding the same charge as the DLC at some voltage V ($C_q = Q_{DLC}/V$). This capacitance derived from (3) and (4) is

$$C_q = C_{i0} + \frac{C_{i1}}{2} \times V \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

with $Q_{DLC} = \int C_{diff}\, dV = \int (C_{i0} + C_{i1} \times V) \times dV = C_{i0} \times V + (1/2) C_{i1} \times V^2$

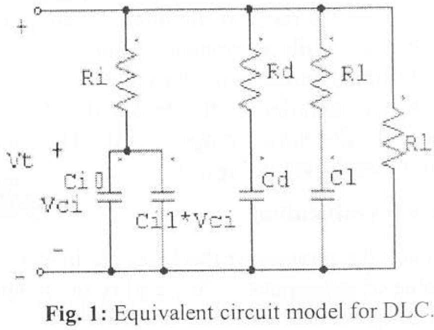

Fig. 1: Equivalent circuit model for DLC.

Second, the equivalent capacitance defined as the capacitance value of a linear capacitor holding the same energy as the DLC at some voltage. This capacitance derived from (6) is

$$C_e = C_{i0} + \frac{2}{3} C_{i1} \times V \ldots\ldots\ldots\ldots\ldots (7)$$

Immediate Branch Parameters Identification

The immediate branch parameters are identified charging a fully discharged DLC with high constant current. As the time constant of the immediate branch is small compared with the time constant of the other two branches, it is assumed that all the charge is initially stored in the immediate branch.

Event n = 0:

Before identification $V_0 = 0$ V.

$Q_0 = 0$.

Current source is switched on.

Event n = 1:

$t_1 = 20$ ms.

t_1 is given by the experimental fact that the current rises to the set value in less than 20 ms.

Measure $V_1 = 0.075$V.

After the small time t_1, the DLC terminal voltage is mainly determined by the voltage drop at R_i Parameter:

$$R_i = \frac{V_1}{I_{ch}} = \frac{0.075V}{30A} = 2.5m\Omega$$

Here I_{ch} is 5% of rated current 600A

Event n =2:

Reached when $V_2 = V_1 + \Delta V$.

ΔV is chosen to be 50 mV. V2 = 0.075 + 0.05 = 0.125 V

Measure $t_2 = 0.47$s, $\Delta t = t_2 - t_1 = 0.47 - 0.02 = 0.45$s.

As V_{Ci} is approximately zero, the differential capacitance is equal to C_{i0}

Parameter:

$$C_{io} = I_{ch} \times \frac{\Delta t}{\Delta V} = 30A \times \frac{0.45s}{0.05V} = 270F$$

Event n = 3:

Reached when $V_3 = V_{rated} = 2.3$V.

Measure $t_3 = 37$s

The current source is turned off ($I_{ch} = 0$)

Event n = 4:

$t_4 = t_3 + 20$ ms $= 37$s $+ 0.02$s $= 37.02$s.

20 ms is given by fall time of the current.

Measure $V_4 = 2.25$V

Total charge supplied to the DLC:

$Q_{tot} = I_{ch} \times (t_4 - t_1) = 30A \times (37.02s - 0.02s) = 1110C$

From $Q_{tot} = Cq \times V_4$, we can find Cq = 493 F and then using (6) the voltage-dependent capacitance may be calculated.

$$C_q = C_{i0} + (1/2) C_{i1} \times V_4 \quad \text{or,} \quad C_{i1} = \frac{2}{V_4} \times (C_q - C_{i0})$$

Parameter:

$$C_{i1} = \frac{2}{V_4} \times \left(\frac{I_{ch} \times (t_4 - t_1)}{V_4} - C_{i0} \right) = \frac{2}{V_4} \times \left(\frac{Q_{tot}}{V_4} - C_{i0} \right)$$

$$= \frac{2}{V_4} \times (C_q - C_{i0}) = \frac{2}{2.25V}(493F - 270F) \approx 190\text{F/V}$$

Delayed Branch ParameterS Identification

After event 4, and due to the assumption of distinct time constants, the immediate branch is charged V_4 to and the other two branches are discharged. Then, an internal equivalent current (I_{tr}) flows from the immediate to the delayed branch and the charge redistribution process takes place.

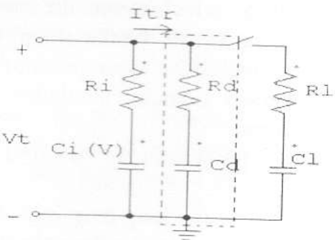

Fig. 2: Delayed branch identification equivalent circuit.

Event n= 5 :

Reached when $V_5 = V_4 - \Delta V = 2.25V - 0.05V = 2.2V$; as

ΔV is chosen to be 50 mV.

Measure $t_5 = 51.03S$, $\Delta t = t_5 - t_4 = 51.03s - 37.02 = 14.01s$

As ΔV is small and C_d is assumed discharged, I_{tr} is virtually constant and given by:

$I_{tr} = (V_4 - \Delta V/2)/R_d$.(R_i neglected because $R_i \langle\langle R_d$)

Relating the transfer current I_{tr} with the change in charge at the immediate branch, we get:

$I_{tr} = C_{diff} \times \Delta V / \Delta t = 693F \times 0.05V / 14.01s = 2.47A$.

C_{diff} is calculated at the voltage

$V_{ci} = V_4 - \Delta V/2 = 2.25V - 0.025V = 2.225V$ using (3)

$C_{diff} = C_{i0} + C_{i1} \times V_{ci} = 270F + 190(F/V) \times 2.225V = 693F$.

Parameter:

$$R_d = \frac{(V_4 - \Delta V/2) \times \Delta t}{C_{diff} \times (\Delta V)} = \frac{2.225V \times 14.01s}{693F \times 0.05V} = 0.88\Omega$$

this is approximated to 0.9Ω

Event n = 6:

$t_6 = t_5 + 3(R_d \times C_d)$.

Typically $R_d \times C_d \approx 100s$.

Measure $V_6 = 1.98V$

As $t_6 - t_5$ is three times the second branch time constant, the charge redistribution from the immediate to the delayed branch has ended at t_6 and $V_{ci} = V_d$ (voltage across delayed branch).

The delayed branch capacitance is calculated using the charge balance:

$Q_{tot} = C_d \times V_6 + V_6 (C_{i0} + C_{i1} \times V_6/2)$

Parameter:

$$C_d = \frac{Q_{tot}}{V_6} - \left(Ci0 + \frac{Ci1}{2} \times V_6 \right)$$

$$= \frac{1110C}{1.98V} - \left(270F + \frac{190(F/V)}{2} \times 1.98V \right) = 102.51\text{F}$$

this is approximated to 100F.

So, t_6 will be $51.03 + 3(0.9\Omega \times 100F) = 321.03s$

Long-Term Branch Parameters Identification

After event 6, the immediate and delayed capacitors are charged to V_6 and the long-term capacitor is fully discharged. After t_6 the charge redistribution from the immediate and delayed branches to the long-term branch takes place. In this case, an internal transfer current (I_{tr}) flows from the first two branches to the long-term branch. The equivalent circuit from Fig.1 represents the situation (fig.3).

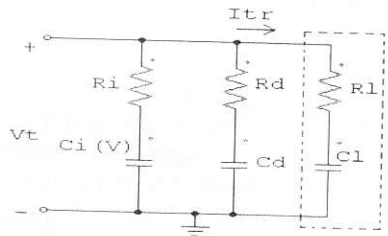

Fig. 3: Long term branch identification equivalent circuit.

Event n = 7:

Reached when $V_7 = V_6 - \Delta V = 1.98V - 0.05V = 1.93V$; as ΔV is chosen to be 50 mV.

Measure $t_7 = 406.28s$, $\Delta t = t_7 - t_6 = 406.28s - 21.03s = 85.25s$.

As ΔV is small and C_l is assumed discharged, I_{tr} is virtually constant and given by:

$I_{tr} = (V_6 - \Delta V/2)/R_l$ (R_i and R_d neglected because $R_i \langle\langle R_d \langle\langle R_l$).

Because R_d is much larger than R_i, the transfer current I_{tr} at this initial instant is supplied mainly from the immediate branch:

C_{diff} is calculated at the voltage $V_{ci} = V_6 - \Delta V/2 = 1.98V - 0.025V = 1.955V$ using (3)

$C_{diff} = C_{i0} + C_{i1} \times V_{ci} = 270F + 190(F/V) \times 1.955V = 641F$.

Parameter:

$$R_l = \frac{(V_6 - \Delta V/2) \times \Delta t}{C_{diff} \times (\Delta V)} = \frac{1.955V \times 85.25s}{641F \times 0.05V} = 5.2\Omega$$

<u>*Event n = 8:*</u> $t_8 = 30$ min.

At t_8 it is assumed that the charge redistribution to the long-term branch has ended and the three equivalent capacitors have the same voltage.

Measure $V_8 = 1.5V$

The long-term capacitance is calculated using the charge balance:

$Q_{tot} = C_l \times V_8 + C_d + V_8(C_{i0} + C_{i1} \times V_8/2)$

Parameter:

$$C_l = \frac{Q_{tot}}{V_8} - \left(C_{i0} + \frac{C_{i1}}{2} \times V_8\right) - C_d$$

$$= \frac{1110C}{V_8} - \left(270F + \frac{190(F/V)}{2} \times 1.5V\right) - 100F$$

$$= 227.5 \text{ F} \approx 220 \text{ F}$$

Leakage Resistance Identification

The leakage resistance is identified by measuring the decrease in the capacitor terminal voltage over a period of 24 hours. The capacitor used for the leakage resistance determination was previously normalized to 2 volts [6]. After the normalization, it is expected that all the internal capacitances in the equivalent model are charged to the same voltage and the voltage decrease as function of time can be attributed to the equivalent leakage resistance. The duration of the test (24 hours) is much greater than the time constants of the three equivalent model branches; therefore, the capacitor is assumed as the parallel equivalent of the three branches and the resultant circuit is an RC circuit. The analysis of a simple RC circuit gives:

$$V_C(t) = V_0 e^{-t/R_{lea}Ct}$$

where C_t is the parallel equivalent capacitance, V_c is the double-layer capacitor terminal voltage and V_o is the initial voltage for the discharge. In the previous equation the value of the leakage resistance is assumed to be much larger than the resistance of the three branches. Using the series approximation for the previous equation with $t << R_{lea} \times C_t$ gives:

$$V_C(t) = V_0(1 - t / R_P C_{lea})$$

Defining ΔV_c as the decrease in terminal voltage after the 24 hours test, the following relation is produced.

$$R_{lea} = \frac{V_0 \Delta t}{\Delta V_C C_t}$$

Table 1: Identified parameter of 470F capacitor

Rated Voltage, $V_{rated} = 2.3V$	
PARAMETER	470F DLC
R_i	2.5mΩ
C_{i0}	270F
C_{i1}	190F/V
R_d	0.9Ω
C_d	100F
R_l	5.2Ω
C_l	220F
R_{lea}	9kΩ

In the previous relation V_o is two volts, ΔV_c is measured after 24 hours found 0.04V [6], C_t is known from the previous identification of the internal capacitances which is equal to 470F and Δt is equal to 24 hours. Hence R_{lea} is approximately 9KΩ. Table 1 shows the average of the Equivalent model parameter values measured for the 470F of double-layer capacitors.

4. Verification Of The Parameters Identification Assumptions

Figure 4 shows the simulation of a simple charge and self charge distribution action. In the figure, the voltages in each of the three equivalent capacitors have been included. The point A shows the instant at which the current source is turned off and the delayed branch begins to be calculated. At this point, the voltage in the delayed branch is lower than 20% of the rated voltage, which means the energy stored in this branch is lower than 4% of the maximum energy in the branch. This result validates the assumption of no charge at the start of the delayed branch calculation used in the parameters identification.

Point B shows the point of the delayed capacitance calculation.

Point C shows the start of the long term branch calculation. In this point the almost equalized voltage in the immediate and delayed branches and the low voltage present in the long term branch at this point. Those two observations confirm the validity of the supposition of independent time behavior of the long term branch with respect to the delayed one.

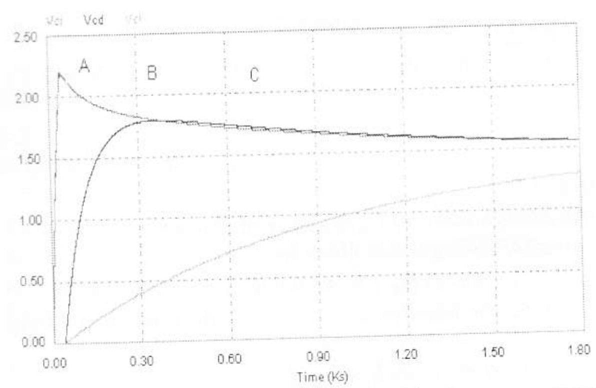

Fig. 4: Verification of the parameter identification assumptions.

5. Verification Of The Proposed Model Using PSIM

The response for 470F DLC was simulated for the equivalent circuit using PSIM [7]. These figures demonstrate a very good agreement between measurements [3, 6] and simulation for a charge cycle using 470 F DLC.

For operating voltages below 1 V or less than 45% of the rated voltage, the error between simulation and experiment increases. The reason for this increased error is due to the assumption of the voltage dependent capacitance in immediate branch. In practice, all the equivalent capacitors have some voltage dependence. The reason for increased error is the energy stored in *RC* branches with very long time constants of hours or days, which have been neglected in the used model.

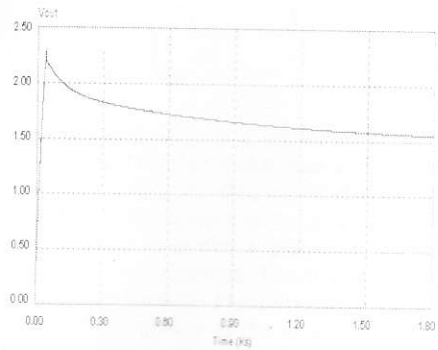

Fig. 5: Simulated output of immediate & delayed branches using PSIM.

It was estimated, that these branches may hold up to 10% of the energy of the DLC. The increased inaccuracies at low voltage or long time spans are in most practical applications of little

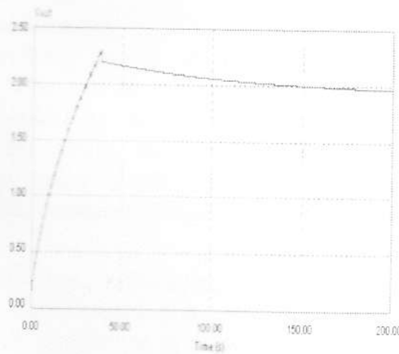

Fig. 6: Simulated output of long term branch.

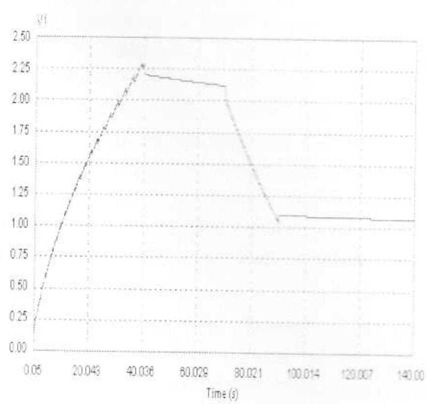

Fig. 7: PSIM Simulated output of a DLC model.

importance, as the capacitor holds little energy at low voltage and is used for short-time energy storage. If these properties matter in specific applications, extended models have to be developed.

6. Conclusion

The studies leading to the simulated practical model of the DLC for applications in power electronics suggest a complex terminal characteristic for this new device. The capacitance of the branch with the fastest response is modeled as a voltage dependent capacitor. RC branches with very long time constants of hours or days have been neglected. The model reflects the internal charge distribution process very well within the considered time span and for voltages above 40% of the rated terminal voltage. At low voltages, the error may reach 10% of the rated voltage because of several assumptions to simplify the model and the parameters' identification. A procedure to determine the required model parameters from terminal measurements has been developed. The model with the measured parameter values has successfully been used to explore the behavior of the DLC. The model provides the desired insight into the complex terminal behavior of the DLC and provides the means to study its application as an energy storage device in power electronic circuits and systems.

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