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Direct Modulation Performance of Quantum Well Semiconductor Laser Diodes Operating in Multimode

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ABSTRACT

A theoretical investigation has been carried out for both the direct sinusoidal modulation and associated noise performance of InGaN based quantum well (QW) semiconductor laser diodes (LDs) operating in multimode. The study is based on the QW lasers with two separate quantum wells with different carrier injection ratios. A model of multimode rate equations is developed by taking into account both symmetric and asymmetric cross-gain saturation. Numerical simulation shows that the mode partition effect exists in both the modulated and unmodulated LDs. From modulation at microwave frequency, highly synchronized oscillation of the modes resulting periodic pulse-like output is observed that contains peaks at modulation frequency and its harmonics. Associated total RIN and modal RIN values suppress with the modulation index. The simulation results have well correspondence with the previously reported theoretical and experimental findings.

Keywords: Direct modulation, Quantum well (QW) InGaN lasers, Semiconductor laser diodes, Rate equation model, Mode partition, Synchronized oscillation, Periodic pulse, Relative intensity noise, LF-RIN.

1. Introduction

Quantum well lasers have sparked a lot of interest due to their numerous benefits like low threshold current density, high modulation rate, and many others [1]. The performance of QW laser diodes like other SLDs when subjected to high-speed modulation is an essential factor for many application areas [2].

Large linewidth enhancement factor and high asymmetric cross-gain saturation contribute to the strong mode coupling in the longer wavelength side [3], such is the case for the 410 nm QW InGaN laser diodes. The strong mode coupling causes simultaneous modes oscillation with random mode fluctuation [4, 5]. Such type of multimode operation increases the relative intensity noise at the laser output [6].

It is well known that sinusoidal modulation of the laser diodes in microwave frequencies near the relaxation oscillation frequency cause noise suppression [7, 8]. With different modulation depths and frequencies, we may get different types of laser output signals like the continuous waveform or the pulsing waveform [9]. These signal waveforms are associated with different types of intensity noise like, the harmonic distortion that causes mode coupling noise or chaos [6, 10, 11]. Bakry *et al*. in [12] characterize the InGaAsP based near-infrared laser diodes under modulation at the resonance frequency and get changes in the mode coupling from anticorrelation to positive correlation with an increase of the modulation depth.

In this paper, we simulate and investigate the mode coupling noise characteristics of the multimode InGaN based quantum well laser diodes under the effect of direct sinusoidal modulation equal to the relaxation oscillation frequency. We model the QW laser diodes through multimode rate equations that consider two separate quantum wells. The rate equation model considers both the symmetric and asymmetric cross-gain saturation terms [6]. The mode coupling strength is calculated by the correlation coefficient of the dominant lasing modes, and the associated noise is characterized by the RIN values of the total output and the three strongest lasing modes. The results help to examine the behaviour of the QW lasers subjected to direct sinusoidal modulation at microwave frequency.

2. Methodology

2.1 Rate Equation Model

We use the extended rate equation model described by L. Uhlig *et al*. [13] to simulate the nonlinear dynamics of the InGaN based quantum well laser diodes. We further include the Langevin noise sources in relation to carrier recombination and spontaneous emission. We consider two separate quantum wells with different carrier injection ratios [14, 15]. Associated carrier numbers at transparency, saturation and threshold for each QW are considered half of the single quantum well.

Following rate equations for the photon number $S_p(t)$ and the carrier number $N_1(t)$ and $N_2(t)$ for two separate wells are used to model the QW semiconductor laser.

$$\frac{dS_p}{dt} = \left(G_{p1} + G_{p2} - G_{th}\right)S_p + \frac{\xi a(N_1+N_2)/V}{\left(2\frac{\lambda_p-\lambda_0}{\delta\lambda}\right)^2+1} + F_{Sp}(t) \quad (1)$$

$$\frac{dN_1}{dt} = -\sum_p A_{p1}S_p - \frac{N_1}{\tau_s} + \chi\frac{I}{e} \quad (2)$$

$$\frac{dN_2}{dt} = -\sum_p A_{p2}S_p - \frac{N_2}{\tau_s} + (1-\chi)\frac{I}{e} \quad (3)$$

In Eqn. (1), G_{p1} and G_{p2} are the gain of the two quantum wells consisting of linear and self- and cross-gain saturation terms and are defined as-

$$G_{p1} = A_{p1} - B_1 S_p - \sum_{q \neq p} \left(D_{pq1} + H_{pq1} \right) S_q \tag{4}$$

$$G_{p2} = A_{p2} - B_2 S_p - \sum_{q \neq p} \left(D_{pq2} + H_{pq2} \right) S_q \tag{5}$$

G_{th} is the threshold gain which depends on both the internal loss and mirror loss. The second term in Eqn. (1) is the spontaneous emission contributing to the lasing modes. The function $F_{Sp}(t)$ representing the Langevin noise source are defined as-

$$F_{Sp}(t) = \sqrt{\frac{V_{SpSp}}{\Delta t}} . g_s \tag{6}$$

where g_s is the random numbers with normal distribution generated to be between -1 and +1.

In Eqns. (2) and (3), the linear gain term A_p of the p-th longitudinal mode is defined as-

$$A_p = \frac{a\xi}{V} \left[N - N_g - bV \left(\lambda_p - \lambda_0 \right)^2 \right] \tag{7}$$

We get two separate equations, A_{p1} for the first well using N_1 and A_{p2} for the second well using N_2. The second term in Eqns. (2) and (3) represents the carrier recombination due to spontaneous emission. The third term is the carrier injection term, and we consider two different injection ratios for the two separate quantum wells to highlight different carrier densities for the different wells. The first well is usually the stronger one being pumped, so we consider χ as $0.5 \leq \chi \leq 1$ [15].

In Eqns. (4) and (5), the second term is the self-saturation with coefficient B characterize the effect of modal photon number on the gain of that lasing mode. The third term is the mode coupling with D_{pq} is the symmetric cross-gain saturation coefficient, and H_{pq} is the asymmetric cross-gain saturation coefficient [16]. This third term characterizes the effect of the other modal photon number on the gain of the lasing mode. These three gain coefficients are defined as-

$$B = \frac{9}{4} \frac{\hbar \omega_p}{\varepsilon_0 n_r^2} \left(\frac{\xi \tau_{in}}{\hbar V} \right)^2 a R_{cv}^2 (N - N_s) \tag{8}$$

$$D_{pq} = \frac{4}{3} \frac{B}{\left(\frac{2\pi c \tau_{in}}{\lambda_p^2} \right)^2 \left(\lambda_p - \lambda_q \right)^2 + 1} \tag{9}$$

$$H_{pq} = \frac{3}{8} \frac{\lambda_p^2}{\pi c} \left(\frac{a\xi}{V} \right)^2 \frac{\alpha \left(N - N_g \right)}{\lambda_q - \lambda_p} \tag{10}$$

We get separate equations for the different quantum wells using N_1, N_2, B_1 and B_2 in Eqns. (8-10). Assuming wavelength of the central mode to be λ_0 with $p = 0$, the other longitudinal modes are defined as-

$$\lambda_p = \lambda_0 + p\Delta\lambda = \lambda_0 + p \frac{\lambda_0^2}{2 n_r L} \qquad p = 0, \pm 1, \pm 2, \ldots \tag{11}$$

Here we consider λ_0 lies at the center of the gain spectrum and neglects any wavelength shift due to other effects [17, 18]. The noise is generated due to the temporal photon number fluctuation, which is defined as $\delta S(t) = S(t) - S_{avg,}$ with $S(t) = \sum S_p$ being the total photon number. The noise content is then defined by the fast Fourier transform (FFT) as [19]-

$$RIN = \frac{1}{S_{avg}^2} \frac{\Delta t^2}{T} \left| FFT[\delta S(t_i)] \right|^2 \tag{12}$$

We include the sinusoidal modulation into the laser rate equation by introducing time-varying current as defined below.

$$I(t) = I_b + I_m cos(2\pi f_m t) \tag{13}$$

In the above equation, I_b is the bias current, I_m is the modulation current, f_m is the modulation frequency and $m = I_m / I_b$ is defined as modulation depth.

2.2 Methods of Simulation

We examine the nonlinear dynamic behaviour of the InGaN based semiconductor laser diodes considering two distinct quantum wells. To do that, we employ numerical simulation of the rate equation model illustrated by the Eqns. (1)-(3). The fourth-order Runge-Kutta method is used to solve the laser rate equations. We consider 13 modes for the simulation with different wavelengths ranging from 409.4 nm to 410.6 nm. The simulation time step is selected to be as small as $\Delta t = 5$ ps to count on relaxation oscillation at a very high frequency. We have done the simulation for a long 2 ms time using the Matlab platform. We use the following parameter values chosen for our numerical simulation so that the rate equations perfectly model the QW InGaN laser diodes.

a = differential gain coefficient = 1.85×10^{-12} m^3/s, b = dispersion parameter of the linear gain spectrum = 3×10^{39} m^{-3}A^{-2}, $\Delta\lambda$ = half-width of spontaneous emission = 20 nm, $|R_{cv}|^2$ = squared value of the dipole moment = 1.48×10^{-58} C^{2}m^2, α = linewidth enhancement factor = 2, ξ = field-confinement factor = 0.2, χ = unequal pumping parameter = 0.7, τ_{in} = intraband relaxation time = 0.01 fs, τ_s = average carrier lifetime = 2 ns, Ns = carrier number characterizing nonlinear gain = 2.01×10^8, Ng = carrier number at transparency = 2.52×10^8, n_r = refractive index of laser active region = 2.6, g_{th} = threshold gain = 2.11×10^{11} [13, 20].

3. Results Analysis

A typical example of the mode dynamics of InGaN quantum well laser diodes are shown in Fig. 1 at injection current $I = 1.8 I_{th}$. Fig. 1(a) shows the modal photon number and gain spectrum are asymmetric. This is due to the relatively high α value we have higher AGS (asymmetric cross-gain saturation). This higher AGS causes the modal gain G_p to be high on the longer wavelength side and low on the shorter wavelength side [4, 5, 16, 21], and the photon density in the longer wavelength side becomes dominant. Due to the higher AGS value, this type of multimode operation with random modal photon density fluctuation with time exists in the InGaN QW laser diodes, as shown in Fig. 1(b). The low ratio (<10) of two higher dominant modes $S_{+10}/S_{+12} = 1.04$ confirms the multimode operation of the QW LDs [22, 23].

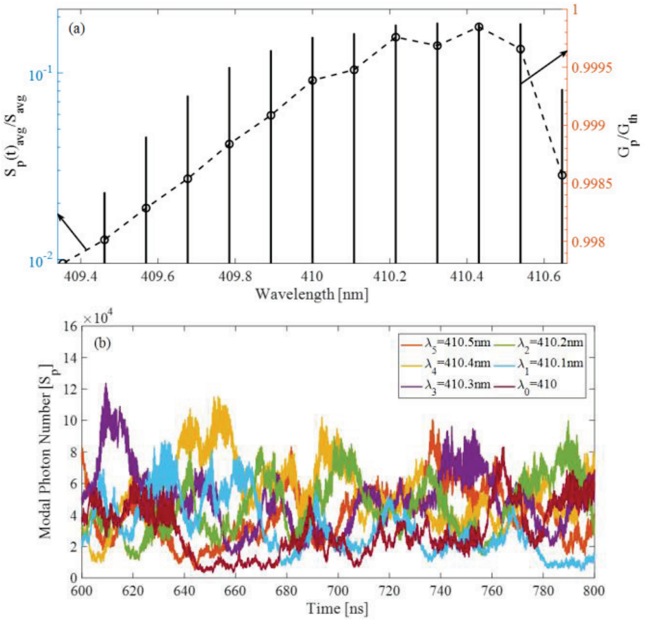

Fig. 1: Mode dynamics of an unmodulated InGaN QW laser diode at input current $I = 1.8I_{th}$. (a) Time-averaged modal gain G_p normalized to threshold gain G_{th} and photon intensity $\overline{S_p(t)}$ of each mode normalized to average photon intensity $\bar{S}$ in the laser output $S(t)$. (b) Time variation photon numbers of five dominant modes.

Fig. 1(b) shows the time variation of the modal photon number of the six dominant modes, namely- λ_0 to λ_{+5} for 0.2 ms with an arbitrary time range of 600 ~ 800 ns. This type of multimode operation with random photon fluctuation may be originated from the high correlation value, originated due to higher AGS value, between the mode-couple. The correlation coefficients between the modes can be calculated from [6]-

$$C_{p,q}(\tau) = \frac{\langle \delta S_p(t)\delta S_p(t+\tau)\rangle}{\langle S_p\rangle\langle S_q\rangle} \tag{19}$$

with $\delta S_p(t) = S_p(t) - \bar{S}$.

The correlation coefficients measured among several mode-couples, namely- $C_{+3,+4}$, $C_{+2,+3}$, $C_{+4,+5}$, and $C_{0,+1}$ vary around 0.92 ~ 0.99. Multimode operation is originated from these comparatively stronger correlation values among several dominant modes, as confirmed from Fig. 1(b). The opposite type of strong anticorrelation coefficients was found by M. Ahmed *et al.* [22] for the InGaAsP based near-infrared laser diodes where they got periodic mode competition among three dominant modes rather than multimode operation with random photon number fluctuation. Others got a stable single-mode operation in the case of AlGaAs based infrared lasers [3, 23, 24] due to lower α value and weak AGS effect.

Frequency spectra of the laser output and three dominant modes, namely- S_{+3}, S_{+4} and S_{+2}, for both unmodulated and modulated lasers with modulation frequency $f_m = f_r$ (f_r = resonance frequency measured from Fig. 2(a)) and depth of modulation m = 0.8 is presented in Fig. 2. The unmodulated QW InGaN lasers show ~2×10^{-13} Hz^{-1} of total noise in the lower frequency region, and a very high value of ~5×10^{-8} Hz^{-1} LF noise present in the dominant modes indicates the presence of mode partition [25]. After low frequency, the noise of the dominant modes drop sharply but smoothly and depict the characteristic sharp peaks at the resonance frequency f_r = 3.5 GHz. These RIN characteristics correspond well with that of the near-infrared and infrared semiconductor laser diodes, both in theory and experiment [5-7, 26-28]. The RIN spectrum for the modulated LDs shows qualitatively the same characteristics though the total RIN and modal RIN values are reduced by around 20 dB due to the modulation effect. The modulated RIN spectrum is also characterized by the very sharp peaks at the modulation frequency and its harmonics.

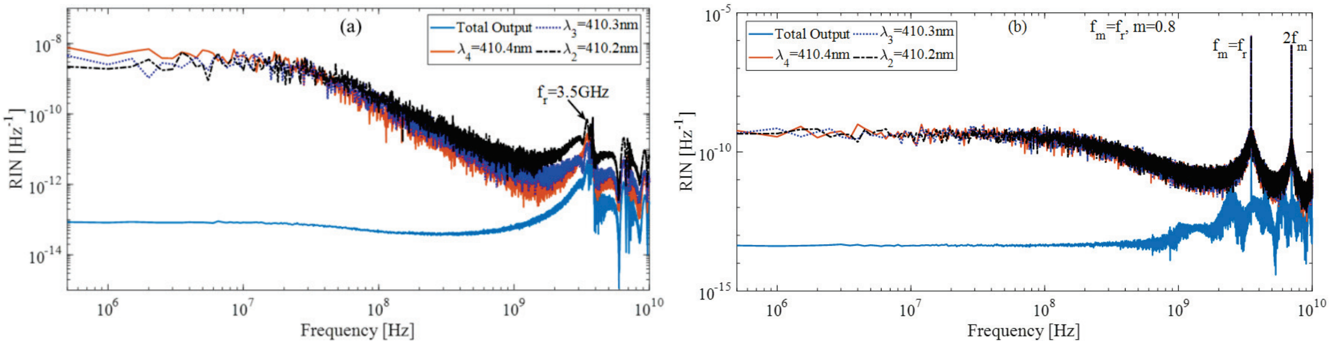

Fig. 2: Spectra of RIN profiles showing relative intensity noise of the total laser output and three dominant modes of the mode number p = +2, +3 and +4 corresponding to the optical wavelength of λ_{+2} = 410.2 nm, λ_{+3} = 410.3 nm and λ_{+4} = 410.4 nm, respectively. (a) RIN profiles of an unmodulated InGaN QW laser. Resonance frequency is measured at f_r = 3.5 GHz. (b) RIN profiles of InGaN QW laser modulated with modulation frequency $f_m = f_r$ and modulation index m = 0.8.

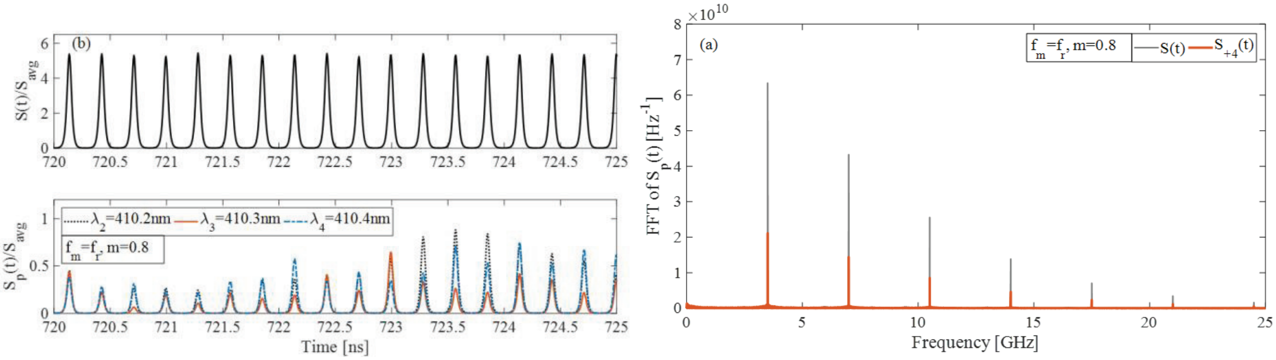

Fig. 3: Mode oscillation characteristics with modulation frequency $f_m = f_r$ = 3.5 GHz, and modulation index m = 0.8. (b) Fourier frequency of total photon number $S = \sum S_p$ and dominant mode S_{+4}. (b) Temporal photon number for the three dominant modes S_{+2}, S_{+3}, S_{+4} and total photon number $S = \sum S_p$, both normalized with time average photon number $\bar{S} = S_{avg}$.

Fig. 3 shows mode oscillation characteristics of the laser output for the modulation index m = 0.8 to represent strong modulation effects on the LDs. The Corresponding Fourier frequency spectrum of the total output and the peak dominant mode is also shown. The photon fluctuations exhibit regular periodic pulses with duration $T_m = 1/f_m$ under the modulation effect. The pulsation of $S(t)$ is originated from the effect of gain switching, combined with the fact that $f_m = f_r$ and there is no chance of the relaxation sub-oscillation to appear [29]. This type of pulsation may also correspond to the spike generation predicted by C. H. Lee *et al*. in [30]. Sharp peaks in the Fourier spectrum at modulation frequency $f_m = f_r$ and its low-order harmonics appear in Fig. 3(b). Hence, it is evident that a large amount of harmonics are present in the laser output signal, which may result distorted output. Also, due to the synchronization of the strong modes oscillation, we get a significant increase in the total photon number at the laser output.

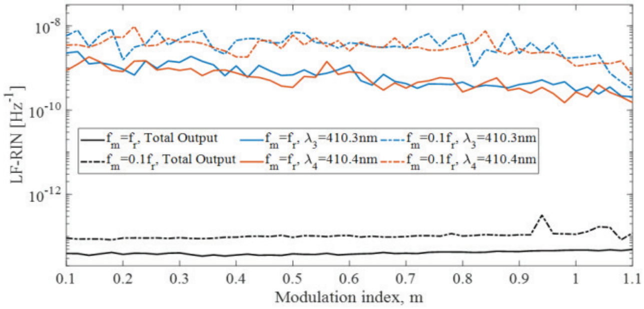

Fig. 4: Variation of RIN values in the lower frequency region with modulation index for the total laser output and two dominant modes, S_{+3} and S_{+4}. Two different modulation frequencies, one at high-frequency modulation $f_m = f_r$ and another at low-frequency modulation $f_m = 0.1 f_r$ are considered here.

Fig. 4 demonstrates the variation of the LF-RIN of the QW InGaN laser diodes modulated with modulation frequency $f_m = f_r$ with the modulation index m = 0.1 ~ 1.1 both for the two strong dominant modes and the total laser output. For comparison, we also plot the corresponding LF-RIN variation for the low-frequency modulation $f_m = 0.1 f_r$ in the figure. The LF-RIN values decrease with the modulation index m, and we get comparatively higher LF-RIN values for the low-frequency modulation than the high-frequency modulation. This outcome agrees with the statement made by M. Ahmed *et al*. in [10] that the sinusoidal modulation works to suppress the associated laser output noise. Significantly higher LF-RIN values for the strong modes compared to the total output confirm the existence of the mode-partition effect [25].

4. Conclusion

InGaN blue-lasers are widely used in laser microscopy, surface scanning, laser printing, data storage and high-frequency optical communication. We, in this paper, demonstrate the mode dynamics of the 410-nm QW InGaN laser diodes under the effect of direct sinusoidal modulation. The unmodulated LDs exhibit multimode operation with random fluctuation due to the strong correlation among several dominant modes. Under the favourable modulation condition, the modulation works to weaken the correlation, and we get regular periodic pulse output originated from the synchronized oscillation of the dominant lasing modes. We get comparatively higher LF-RIN values for the dominant modes than the total output for the modulated and unmodulated laser diodes, confirming the presence of mode partition effect in the quantum well lasers. Direct sinusoidal modulation works to suppress the relative intensity noise, which is confirmed from the fact that the LF-RIN values of the total and stronger modal output decreases with the increase of the modulation index.

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